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# OBTAINING FIBER STRESS–STRAIN CURVES BY RECORDING FIBER VIBRATIONS DURING ELECTRIC-FIELD MEASUREMENTS

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*A simple method for constructing tension curves of thin fibers is proposed which consists of determining the dependence of the frequencies of small transverse vibrations of thin fibers on their tensile strain. The fiber stress at these frequencies is calculated using the classical formula of string vibrations. Instead of force and strain, it is proposed to measure frequency and strain. This method has a number of advantages, in particular, it is more accurate. Resonant frequencies are determined by recording the accompanying electric field resulting from the variation in the charge distributed over the fiber surface. The effectiveness of the method is demonstrated by constructing copolymer fibers of various diameters.*

**Key words:** *fiber, vibration frequency, tension curve, electric signal.*

**Introduction.** To develop and (or) calibrate rheological models of fibers and strings, it is necessary to use stress–strain curves which are determined experimentally using expensive equipment, in particular, high-precision instruments for measuring small forces. In the present paper, we propose a simple and less expensive method for constructing such curves for fibers that admit large strains before fracture. This method is based on measuring the frequencies of small transverse vibrations of a stretched string of such a fiber at specified strain. The required stress–strain relation is then calculated using the theory of small elastic vibrations of a stretched string. Frequencies are measured by recording variations in the electric field induction component by a transducer antenna during transverse vibrations of precharged fibers. This method has a number of advantages. The antenna is located at a distance from the source and, hence, does not introduce errors to he measured values. The method can be used under complicated environmental conditions (severe media and high temperatures), does not require complex equipment, eliminates external noise, and the samples are prepared using conventional techniques. An advantage of the proposed method over direct measurements of forces and displacements is accurate determination of frequencies whereas measuring small forces with a small error is difficult to perform. The method of determining vibration frequencies, modes, and damping is described in [1, 2], along with the results of measurements of transverse vibrations of glass fibers fixed as an elastic cantilever and the observed relationship between the time of charge dissipation from the fiber surface and the degree of damage to this surface.

**1. Experimental Procedure.** The main elements of the experimental setup are an electric field transducer (antenna) and loading and recording devices. The loading device has two metal clamps with dielectric liners for fastening fibers, one of which is immovable and the other can be moved horizontally by means of a rod. The transducer and clamps are placed in a 162 × 89 × 32 mm brass box with a wall thickness of 7 mm to provide screening of external fields. The transducer was a copper wire rod (diameter 2 mm and length 10 mm) which was placed at a distance of several millimeters from the fiber — a capacitive transducer (intrinsic capacitance 1–2 pF), which recorded only electric field changes. The transducer was connected to a LeCroy WaveSurfer 422 double-beam digital oscilloscope through oscillographic probes (active resistance 10 MΩ, input capacitance 15–35 pF), which provided transmission of undistorted spectra of signals in a frequency band of 1–400 kHz. The signals were

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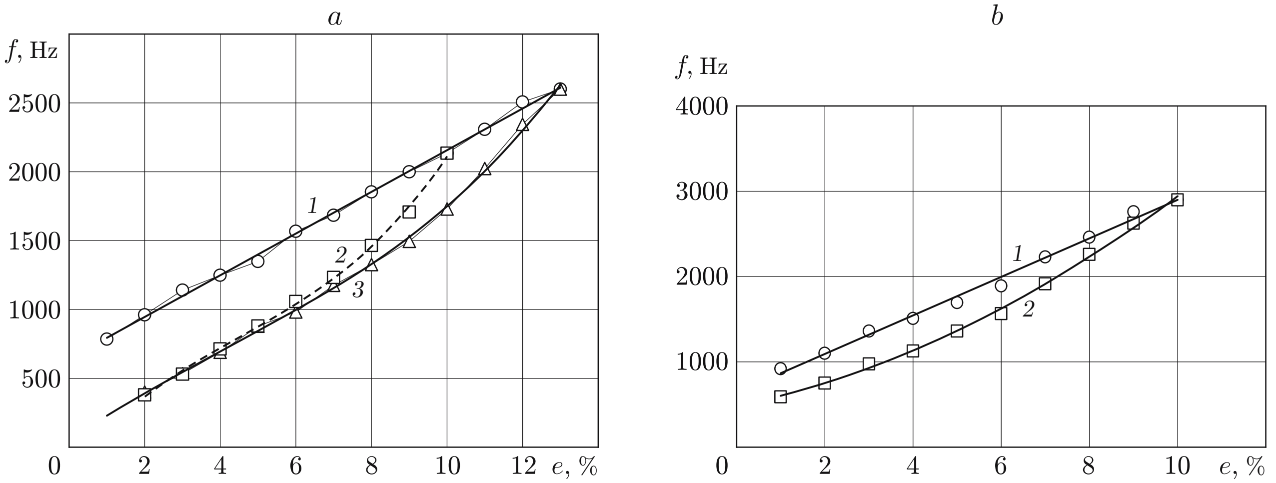

Fig. 1. Frequency versus strain in fibers of various diameters under loading and unloading: $d =$ 140 (a) and =100 $\mu$m (b); curve 1 refers to loading and curves 2 and 3 refer to unloading from the state of tension for $e = 10$ (2) and 13% (3).

recorded without preliminary amplification. The oscilloscope was triggered by the leading edge of the recorded signals upon reaching the specified signal amplitude. The results of testing of the procedure are described in detail in [1].

Previously, it has been established that, without preliminary application of an electric charge on the fiber surface, fiber vibrations are not recorded. In the presence of a positive or negative charge, the period of the first-mode vibration is determined with high accuracy by oscillograms. One also observes the superposition of vibrations of the next modes (for asymmetric excitation), the rotation of the plane of vibrations, and their damping, which is determined by the damping coefficient [1, 2]. The effect of the value and sign of the charge has not been found.

We transform the classical formula for the first natural frequency of a string:

$$f = \frac{1}{2L}\sqrt{\frac{F}{m/L}} = \frac{1}{2L}\sqrt{\frac{FL}{\rho_0 S_0 L_0}} = \frac{1}{2\sqrt{L}}\sqrt{\frac{\sigma}{\rho_0 L_0}} = \frac{1}{2L_0}\sqrt{\frac{\sigma}{\rho_0(1+\varepsilon)}}. \qquad (1)$$

Here $F$ is the tension force, $m$, $L_0$, $L$, $\rho_0$, and $S_0$ are the mass, initial and current lengths, initial density, and area of the string, respectively, $\varepsilon$ is the strain of the string, and $\sigma$ is the nominal normal stress. Relation (1) implies that

$$\sigma = 4f^2 L_0^2 \rho_0 (1+\varepsilon), \qquad (2)$$

where $\varepsilon = (L - L_0)/L_0$ and $\sigma = F/S_0$. Determining the dependence of the natural frequency of the stretched fiber on its strain $f(\varepsilon)$ and substituting the result into formula (2), we obtain the relation $\sigma(\varepsilon)$.

**2. Results of Experiments.** The experiments were performed with samples of Nikko Vexter copolimer fibers with a diameter $d = 60$, 100, and 140 $\mu$m, $L_0 = 100$ mm, and $\rho_0 = 1.14$ g/cm$^3$. A surface negative or positive charge was applied onto the sample. The value of the charge was important only for reliable signal recording. In the experiment, the sample was extended, so that the strain sequentially took values $e = 100\varepsilon = 2\%, 3\%, \ldots$ . For each strain, we measured the first natural frequency of string vibrations caused by pinching the center of the string with a wooden toothpick. Once the strain reached a maximum value close to the fracture strain $e_c$, unloading in the reverse order began. After that, the samples changed. The frequency depended only slightly on the amplitude (within a 2% spread of the experimental data), the length of the measurement region (number of periods) during significant vibration damping, and on the surface charge on the fiber, but it depended significantly on the fiber diameter (the scale effect was substantial). The results of the experiment were reproducible (within the indicated spread) although the time steps — the time intervals after recording of specified strain or between acts of vibrations — varied from several seconds to several minutes. This is due to the fact that, at strains smaller (by more than 2%) than the fracture strain $e_c$, the stress relaxation of fiber of this type is incomplete and occurs rapidly (within a few seconds); therefore, the obtained results actually correspond to long-term loading.

TABLE 1

Coefficients of Approximating Polynomials versus Fiber Diameter

| $d$, $\mu$m | $A$ | $B$ | $C$ | $D$ | $E$ | $G$ |
|---|---|---|---|---|---|---|
| 140 | 643.5 | 151.3 | −164.0 | 330.3 | −38.75 | 2.851 |
| 100 | 513.1 | 183.0 | 58.0 | 327.4 | −38.37 | 2.858 |
| 60 | 639.3 | 226.0 | 479.1 | 109.0 | 13.73 | 0 |

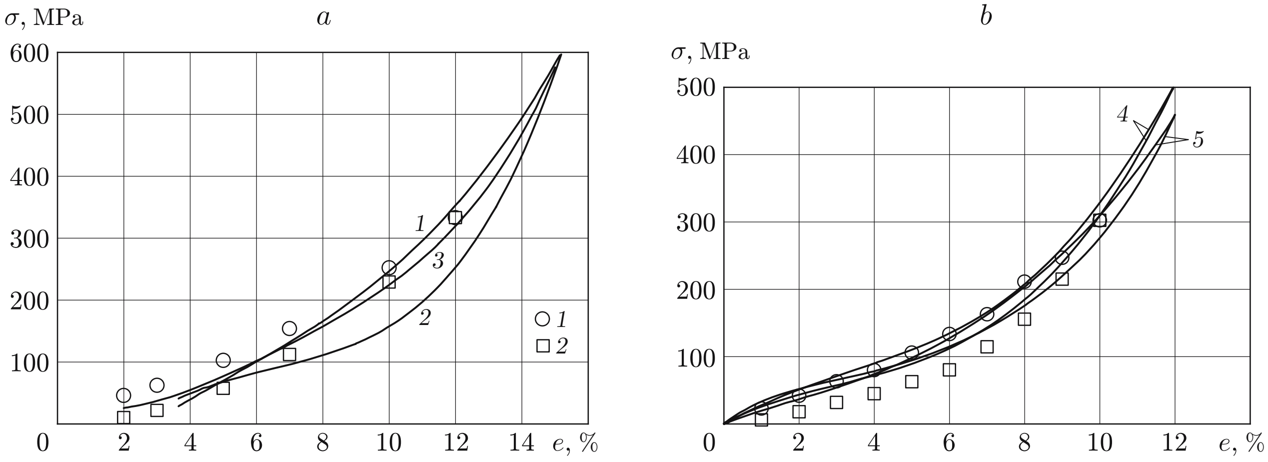

Fig. 2. Curves of long-term loading and unloading for fibers of various diameters: $d = 140$ (a) and 100 $\mu$m (b); curves 1–3 refer to the results of direct measurements for loading (1), unloading (2), and repeated loading (3); curves 4 and 5 refer to the data of experiments with loading and unloading, respectively; points 1 and 2 refer to the calculation results obtained by formula (1) for loading and unloading, respectively.

Figure 1 gives experimental points and their approximating curves of frequency versus fiber strain under rigid loading and unloading (the approximating curves are plotted using the Excel program). The points corresponding to the loading process are on the straight line $f = A + Be$. The curve corresponding to unloading from the state $e = 10\%$ is approximated by the third-order polynomial $f = C + De + Ee^2 + Ge^3$. The values of the coefficients $C$, $D$, $E$, and $G$ depend on fiber diameter (see Table 1): as the diameter decreases, the slope of the straight lines also increases, causing a scale effect with respect to the curves of $\sigma(\varepsilon)$ calculated by Eq. (2). For a fiber of diameter $d = 140$ $\mu$m, Fig. 1a gives the approximating curve that corresponds to unloading from the state $e = 13\%$: $f = 43.38 + 195.3e - 11.6e^2 + 0.9141e^3$. As the strain decreases, this curve asymptotically approaches the curve corresponding to unloading from the state $e = 10\%$. Similar frequency measurements were performed with string of halved length ($L_0 = 50$ mm). The condition of proportionality of the ratio of the frequencies to these lengths, which follows from formula (1), was satisfied with adequate accuracy.

Figures 2 and 3 show curves of $\sigma(e)$ calculated by formula (1). For comparison, the same figures gives results of direct long-term measurements of the tensile force and fiber length increment, which were then converted to the relation $\sigma(e)$. In the experiments, we used a ST-02 electronic dynamometer with a measurement error of ±2.5 g. All unloading curves are below the loading curves, and the difference between them is the larger the greater the maximum strain reached. Hysteresis is due to viscosity and the fiber degradation caused by a change in its internal structure at large strains. In Fig. 2a, the intermediate curve 3 corresponds to repeated loading. The spread of the measured $\sigma$ and $e$ for fiber samples of diameter $d = 60$ and 140 $\mu$m is significant, but, generally speaking, measurements of the mechanical characteristics of polymer fibers have always yielded a large statistical spread [4]. For a fiber of diameter $d = 100$ $\mu$m, this spread was insignificant. Figure 2b shows loading–unloading curves obtained in two experiments by direct measurements of stress and strain. It is evident that the experimental data are in good agreement with each other. The mechanical properties of the fiber of diameter $d = 100$ $\mu$m turned out to be better than those of the tested fibers of other diameters, the measurement results obtained by different methods being in satisfactory agreement. The greatest spread in the results of direct measurements was observed

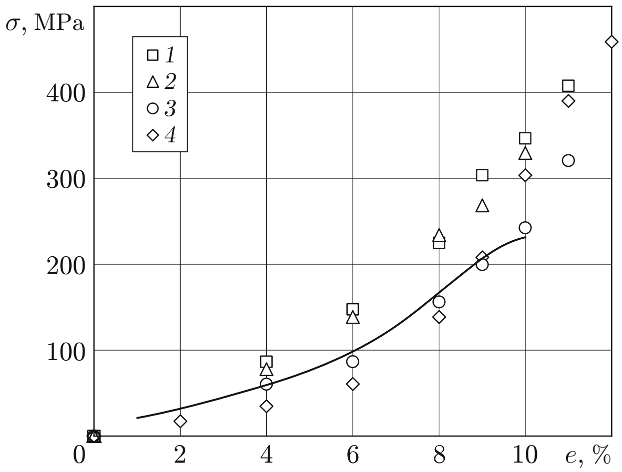

Fig. 3. Experimental data of long-term tension tests for a thin fiber 60 $\mu$m in diameter (points) and a loading curves plotted from frequency measurement data (curve) for $L_0 = 582$ (1), 608 (2), 630 (3), and 692 mm (4).

in tests of a thin fiber of 60 $\mu$m diameter. Nevertheless, the tension curve plotted from the results of frequency measurements (see Fig. 3) lies in the confidence interval of the spread of the direct measurement results obtained in four tension experiments with this fiber at $L_0 = 692$, 630, 608, and 582 mm and a time step of 30 sec.

**Conclusions.** The data given in the present work confirm the effectiveness of using the vibration recording method to obtain tension curves of thin fibers, capable of sustaining large strains before fracture. This method (unlike direct measurements of force and displacement) measures the frequencies of transverse vibrations and elongation of fibers in tension. These quantities are then converted to stress–strain relations. Performing experiments with various exposures between strain steps, it is possible to obtain information on relaxation processes in fibers as a function of the degree of fiber strain and to construct tension and unloading curves under the action of external factors (temperature, aggressive media, and physical fields) [4] influencing the mechanical properties of fibers when direct measurements are difficult to perform. The proposed method is promising for obtaining the required mechanical characteristics of microsized and nanosized solids since measurements of resonant frequencies are usually more accurate than measurements of small forces (an adequate accuracy of direct measurements is reached using expensive equipment).

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